

Dynamical Cosmic Vacuum against a rigid Cosmological Constant

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In the centenary of the introduction of the cosmological constant, Λ , by Einstein in his gravitational field equations, and after about two decades of the first observational papers confirming the accelerated expansion of the universe, we are still facing the question whether the cause of it is a rigid Λ -term or a mildly evolving dynamical dark energy (DE). In this work we perform an overall fit to the SNIa+BAO+ $H(z)$ +LSS+CMB data through the Λ CDM parametrization along with a triad of dynamical vacuum models (DVMs) in interaction with dark matter. We find clear signs (at $> 3.3\sigma$ c.l.) of dynamical DE with the Λ CDM. We also find that two of the DVMs stand out very significantly, to the extent that the traditional $\Lambda = \text{const.}$ picture becomes excluded at an unprecedented $\sim 4\sigma$ c.l. This conclusion is strongly supported by Akaike and Bayesian criteria.

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Introduction: The Λ -term in Einstein's field equations has been there for slightly more than 100 years [1], but we still ignore its ultimate theoretical nature. Meanwhile, we observe that the universe is in accelerated expansion [2, 3], and the simplest hypothesis for it has been to assume that Λ is a nonvanishing and positive constant. This is the firm point of view advocated by the concordance or Λ CDM model, and is supported by numerous observations [4]. However, Λ harbors one of the most profound (and unsolved) theoretical enigmas of fundamental physics: the cosmological constant (CC) problem [5, 6], namely the preposterous mismatch between the typical prediction for Λ in quantum field theory (QFT) – e.g. in the standard model of particle physics – and the measured value from cosmological observations [2–4]. Clearly, we do not have a good control on the theoretical status of Λ at present. For this reason Λ has been promoted into the multifarious concept of dark energy (DE) [7]. For example, scalar field models have been proposed since long ago either to adjust dynamically the value of the vacuum energy (e.g. the cosmon model [8]) or to explain the coincidence problem with the notion of quintessence etc [9–12], among many other alternatives, see [6, 7] and references therein. Whether Λ (or some appropriate DE ersatz) is truly a constant or not is a matter that at this point should be settled empirically. This means to compare efficiently the Λ CDM ability to describe the bulk of the cosmological observations with other models in which Λ , or equivalently the vacuum energy density $\rho_\Lambda = \Lambda/(8\pi G)$ (G being Newton's gravitational coupling), is a mild dynamical function of, say, the scale factor, $a(t)$, or the Hubble rate, $H(t) = \dot{a}(t)/a(t)$, both

evolving with the cosmic time. There are indeed theoretically motivated dynamical vacuum models (DVMs) of this kind in the market (cf. e.g. the reviews [13, 14]), and one would like to know if the available observational data has a real preference for some of them as compared to the Λ CDM. We devote this work to show that there exist DVMs capable of seriously challenging the $\Lambda = \text{const.}$ hypothesis when faced to a large set of significant cosmological observations.

DVMs: We start from a generic cosmological framework described by the spatially flat Friedmann-Lemaître-Robertson-Walker metric, in which matter is in interaction with vacuum energy density ρ_Λ (with pressure $p_\Lambda = -\rho_\Lambda$). We assume $G = \text{const.}$ but $\dot{\rho}_\Lambda \neq 0$. The Friedmann and acceleration equations read formally identical to the standard Λ CDM case:

$$3H^2 = 8\pi G (\rho_m + \rho_r + \rho_\Lambda(\zeta)) \quad (1)$$

$$3H^2 + 2\dot{H} = -8\pi G (p_r - \rho_\Lambda(\zeta)), \quad (2)$$

where ρ_r is the radiation component ($p_r = \rho_r/3$) and $\rho_m = \rho_b + \rho_{dm}$ involves the contributions from baryons and cold dark matter (DM). Finally, in our case we assume that $\rho_\Lambda(\zeta)$ is a function of a cosmic variable $\zeta = \zeta(a)$, which ultimately depends on the scale factor or the cosmological redshift, $z = a^{-1} - 1$. The local conservation law associated to the above equations reads:

$$\dot{\rho}_r + 4H\rho_r + \dot{\rho}_m + 3H\rho_m = Q, \quad (3)$$

where $Q = -\dot{\rho}_\Lambda$ is the source. In the concordance model, $Q = 0$ since $\rho_\Lambda = \text{const.}$ Here we go a step further and focus on three types of DVMs with $0 < |Q| \ll \dot{\rho}_m$. We assume that radiation and baryons are self-conserved, so their energy densities evolve in the standard way, i.e. $\rho_r(a) = \rho_{r0} a^{-4}$ and $\rho_b(a) = \rho_{b0} a^{-3}$. The possible dynamics of ρ_Λ is exclusively associated to the exchange of energy with the DM, and hence we can rewrite (3) as follows:

$$\dot{\rho}_{dm} + 3H\rho_{dm} = Q, \quad \dot{\rho}_\Lambda = -Q. \quad (4)$$

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Model	h	$\omega_b = \Omega_b h^2$	n_s	Ω_m	ν_i	w	χ^2_{\min}/dof	ΔAIC	ΔBIC
Λ CDM	0.692 ± 0.004	0.02253 ± 0.00013	0.976 ± 0.004	0.296 ± 0.004	-	-1	84.88/85	-	-
XCDM	0.672 ± 0.007	0.02262 ± 0.00014	0.976 ± 0.004	0.311 ± 0.007	-	-0.923 ± 0.023	74.08/84	8.55	6.31
RVM	0.677 ± 0.005	0.02231 ± 0.00014	0.965 ± 0.004	0.303 ± 0.005	0.00158 ± 0.00042	-1	69.72/84	12.91	10.67
Q_{dm}	0.678 ± 0.005	0.02230 ± 0.00015	0.965 ± 0.004	0.302 ± 0.005	0.00216 ± 0.00060	-1	70.50/84	12.13	9.89
Q_Λ	0.691 ± 0.004	0.02230 ± 0.00016	0.966 ± 0.005	0.298 ± 0.005	0.00601 ± 0.00253	-1	79.22/84	3.41	1.17

TABLE I: The best-fit values for the Λ CDM, XCDM and the three DVMs, including their statistical significance (χ^2 -test and Akaike and Bayesian information criteria, AIC and BIC). The ΔAIC and ΔBIC increments clearly favor the dynamical DE options. In particular, the RVM and Q_{dm} are strongly favored. Our fit is based on a rich and fully updated SNIa+BAO+ $H(z)$ +LSS+CMB data set: namely 31 points from the JLA sample of SNIa [18], 11 from BAO [19–24], 30 from $H(z)$ [25–31], 13 from linear growth [22, 32–40], and 4 from CMB [41]. See the companion paper [42] for a detailed discussion of the data and a comprehensive exposition. The specific fitting parameter for each DVM is $\nu_i = \nu$ (RVM), ν_{dm} (Q_{dm}), ν_Λ (Q_Λ), whilst for the XCDM is the EoS parameter w . Its value is fixed at -1 for the DVMs. The remaining parameters are standard ($h, \omega_b, n_s, \Omega_m$). The number of degrees of freedom (dof) is equal to the number of data points minus the number of fitting parameters (4 for the Λ CDM and 5 for the DVMs and the XCDM). The parameter M in the SNIa sector [18] was dealt with as a nuisance parameter and has been marginalized over analytically.

One of the models under study is the so-called running vacuum model (RVM), which can be motivated in the context of QFT in curved space-time (see [13, 14] and references therein). In this case, the cosmic variable ζ can be identified not just with the scale factor but with the full Hubble rate: $\zeta = H$. This special theoretical status of the RVM as compared to the other models considered here will have positive phenomenological implications. The model can actually be extended to provide an effective description of the cosmic evolution starting from the early universe up to our days [13–16]. Well after inflation and up to our days, the vacuum energy density in the RVM can be written in the relatively simple form:

$$\rho_\Lambda(H) = \frac{3}{8\pi G} (c_0 + \nu H^2). \quad (5)$$

The additive constant $c_0 = H_0^2 (\Omega_\Lambda - \nu)$ is fixed by the boundary condition $\rho_\Lambda(H_0) = \rho_{\Lambda 0}$, where $\rho_{\Lambda 0}$ and H_0 are the current values of these quantities, and Ω_Λ is the present vacuum density parameter. The dimensionless coefficient ν encodes the dynamics of the vacuum at low energy and can be related with the β -function of the running of ρ_Λ . Thus, we naturally expect $|\nu| \ll 1$. An estimate of ν in QFT indicates that it is of order 10^{-3} at most [17], but here we will treat it as a free parameter and hence we shall deal with the RVM on pure phenomenological grounds, thus fitting ν to the observational data (cf. Table I and Fig. 1).

In the RVM, the source function Q in (4) is not just put by hand (as in the case of the *ad hoc* models we will introduce in a moment). It is a calculable expression from (5) and Friedmann’s equation. We find:

$$\text{RVM:} \quad Q = -\dot{\rho}_\Lambda = \nu H(3\rho_{dm} + 3\rho_b + 4\rho_r). \quad (6)$$

If baryons and radiation would also interact with vacuum, their densities in Eq. (6) would not follow the standard conservation laws and the analysis would be different [43–45].

Next we include in our study two phenomenological DVMs in which the source function Q is introduced by hand, i.e. without any special theoretical motivation.

Two possible ansatzs are the following:

$$\text{Model } Q_{dm}: \quad Q_{dm} = 3\nu_{dm}H\rho_{dm} \quad (7)$$

$$\text{Model } Q_\Lambda: \quad Q_\Lambda = 3\nu_\Lambda H\rho_\Lambda. \quad (8)$$

Model Q_Λ was previously studied in [46], but as we shall see we do not agree with their analysis, see also [47]. Model Q_{dm} was recently studied in [48]. It is closer to the RVM than Q_Λ , but certainly not identical, confer equations (6)–(8).

The dimensionless coefficients $\nu_i = (\nu, \nu_{dm}, \nu_\Lambda)$ for each model (RVM, Q_{dm} , Q_Λ) parameterize the evolution of the vacuum energy density and the strength of the dark-sector interaction. For $\nu_i > 0$ the vacuum decays into DM (which is thermodynamically favorable) whereas for $\nu_i < 0$ is the other way around.

The energy densities of the DVMs can be computed straightforwardly. For simplicity we only quote the leading parts both for the dark matter densities

$$\begin{aligned} \text{RVM: } \rho_{dm}(a) &= \rho_{dm0} a^{-3(1-\nu)} + \rho_{b0} (a^{-3(1-\nu)} - a^{-3}) \\ Q_{dm}: \rho_{dm}(a) &= \rho_{dm0} a^{-3(1-\nu_{dm})} \\ Q_\Lambda: \rho_{dm}(a) &= \rho_{dm0} a^{-3} + \frac{\nu_\Lambda}{1-\nu_\Lambda} \rho_{\Lambda 0} (a^{-3\nu_\Lambda} - a^{-3}), \end{aligned} \quad (9)$$

and for the corresponding vacuum energy densities:

$$\begin{aligned} \text{RVM: } \rho_\Lambda(a) &= \rho_{\Lambda 0} + \frac{\nu \rho_{dm0}}{1-\nu} (a^{-3(1-\nu)} - 1) \\ Q_{dm}: \rho_\Lambda(a) &= \rho_{\Lambda 0} + \frac{\nu_{dm} \rho_{dm0}}{1-\nu_{dm}} (a^{-3(1-\nu_{dm})} - 1) \\ Q_\Lambda: \rho_\Lambda(a) &= \rho_{\Lambda 0} a^{-3\nu_\Lambda}. \end{aligned} \quad (10)$$

In the numerical analysis, however, we include the full expressions, see [42] for details. For $\nu_i \rightarrow 0$ we recover, of course, the Λ CDM. The Hubble function for each model can easily be obtained from these formulas using Friedmann’s equation. It will be convenient to fit also the data through the simple XCDM parameterization of the dynamical DE [49]: $\rho_X(a) = \rho_{X0} a^{-3(1+w)}$, with $\rho_{X0} = \rho_{\Lambda 0}$, where w is the (constant) equation of state (EoS) parameter of the generic DE entity X . For $w = -1$ it boils down to the CC term. The XCDM fit will be useful to roughly mimic a (non-interactive) DE scalar field with

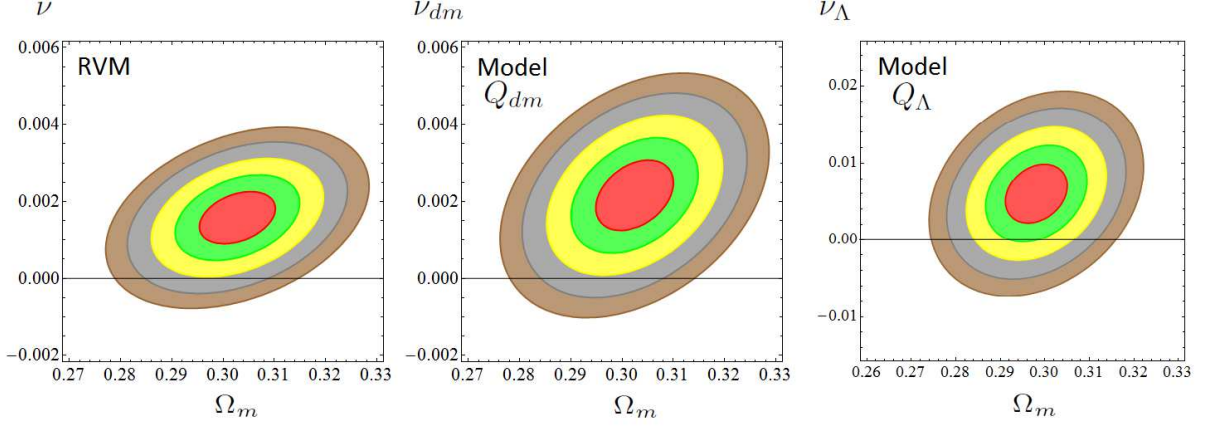


FIG. 1: Likelihood contours for the DVMs in the (Ω_m, ν_i) plane for the values $-2 \ln \mathcal{L}/\mathcal{L}_{max} = 2.30, 6.18, 11.81, 19.33, 27.65$ (corresponding to $1\sigma, 2\sigma, 3\sigma, 4\sigma$ and 5σ c.l.) after marginalizing over the rest of the fitting parameters indicated in Table I. We estimate that for the RVM, 94.80% (resp. 89.16%) of the 4σ (resp. 5σ) area is in the $\nu > 0$ region. For the Q_{dm} we find that 95.24% (resp. 89.62%) of the 4σ (resp. 5σ) area is in the $\nu_{dm} > 0$ region. Finally, for the Q_Λ we estimate that 99.45% (resp. 90.22%) of the 2σ (resp. 3σ) area is in the $\nu_\Lambda > 0$ region. Subsequent marginalization over Ω_m increases slightly the c.l. and renders the fitting values indicated in Table I. The Λ CDM ($\nu_i = 0$) appears disfavored at $\sim 3.8\sigma$ c.l. as compared to the RVM, at $\sim 3.6\sigma$ c.l. versus model Q_{dm} and at $\sim 2.4\sigma$ c.l. with respect to model Q_Λ .

constant EoS. One can also fit the same data assuming a slowly evolving EoS, e.g. within the CPL parametrization [50]; and also using a realistic quintessence model, such as the original Peebles & Ratra model [9]. The results are presented in [42] and [51], and they consistently lead to significant signs of dynamical DE compatible with the current analysis.

Fitting analysis: The analysis of the LSS data plays a crucial role and deserves some remarks. In the presence of dynamical vacuum the matter density contrast $\delta_m = \delta\rho_m/\rho_m$ obeys the equation [42, 43]:

$$\ddot{\delta}_m + (2H + \Psi) \dot{\delta}_m - (4\pi G\rho_m - 2H\Psi - \ddot{\Psi}) \delta_m = 0, \quad (11)$$

where $\Psi \equiv Q/\rho_m$, and Q for each DVM is given by Eqs. (6)-(8). For $\rho_\Lambda = \text{const.}$ and for the XCDM, $Q = 0$, and Eq. (11) reduces to the Λ CDM form. Such equation ensues from the covariant law $\nabla_\nu T^{\mu\nu} = Q^\mu$ and the source 4-vector $Q^\mu = QU^\mu$, where U^μ is the matter 4-velocity, and $T^{\mu\nu}$ is the ordinary energy-momentum tensor for the matter fluid. If $\vec{v} = \vec{\nabla} v_m$ is the associated peculiar velocity, with potential v_m , one can show that the relation $\delta\rho_\Lambda = aQv_m$ is automatically satisfied and the usual Euler equation preserved [42]. To solve Eq. (11) we have to fix the initial conditions for δ_m and $\dot{\delta}_m$ for each model at high redshift, say at $z_i \sim 100$ ($a_i \sim 10^{-2}$), when non-relativistic matter dominates both over radiation and the vacuum contribution, see the details of the procedure in [42].

Equipped with the above generalized matter perturbations equation and the appropriate initial conditions, the analysis of the linear LSS regime is implemented with the help of the weighted linear growth $f(z)\sigma_8(z)$, where $f(z) = d \ln \delta_m / d \ln a$ is the usual growth and $\sigma_8(z)$ is the rms mass fluctuation amplitude on scales of $R_8 = 8 h^{-1}$

Mpc at redshift z . Such amplitude reads [42]:

$$\sigma_8(z) = \sigma_{8,\Lambda} \frac{\delta_m(z)}{\delta_{m\Lambda}(0)} \sqrt{\frac{\int_0^\infty k^{n_s+2} T^2(k, \vec{q}) W^2(kR_8) dk}{\int_0^\infty k^{n_{s,\Lambda}+2} T^2(k, \vec{q}_\Lambda) W^2(kR_{8,\Lambda}) dk}}, \quad (12)$$

where W is a top-hat smoothing function. Apart from the spectral index, n_s , the remaining four fitting parameters are collected in the vector $\vec{q} = (h, \omega_b, \Omega_m, \nu_i)$ and are involved in the transfer function $T(k, \vec{q})$ for the various models (including the Λ CDM, with $\nu_i = 0$; and the XCDM, with w replacing ν_i). We take such function as given in [52]. Similarly, $n_{s,\Lambda}$ and $\vec{q}_\Lambda = (h_\Lambda, \omega_{b,\Lambda}, \Omega_{m,\Lambda}, 0)$ stand for the fixed parameters of the fiducial model, which we use in order to define the normalization of the power spectrum. We take for it the Λ CDM at fixed parameter values from the Planck 2015 TT,TE,EE+lowP+lensing analysis [4]. See [42] for further explanations.

In Fig. 2 we display $f(z)\sigma_8(z)$ for the various models using the fitted values of Table I. We also depict the curve for model Q_Λ under the conditions of Ref. [46]. We disagree both in magnitude and sign concerning their fitted parameter q_V ($\equiv 3\nu_\Lambda$ in our notation). We find $q_V > 0$ whereas they find $q_V < 0$. Their negative sign implies that the corresponding DM mass density in (9) eventually becomes negative, which is odd. For models Q_Λ and Q_{dm} our results are compatible with those of [47, 48], but in our case we were able to attain much higher accuracy, allowing us to claim significant hints of physics beyond the Λ CDM (see below).

For the model comparison in Table I we have defined a joint likelihood function \mathcal{L} . Assuming Gaussian errors, the total χ^2 to be minimized reads:

$$\chi_{tot}^2 = \chi_{SNIa}^2 + \chi_{BAO}^2 + \chi_H^2 + \chi_{LSS}^2 + \chi_{CMB}^2. \quad (13)$$

Each one of these terms is defined in the standard way

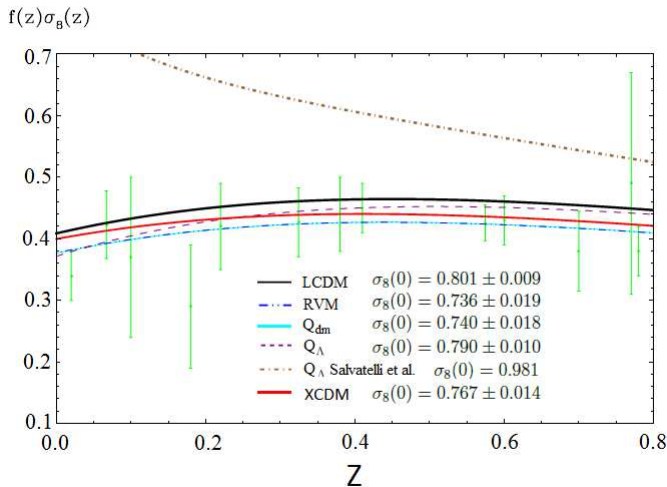


FIG. 2: The LSS data (i.e. $f(z)\sigma_8(z)$, cf. caption of Table I) and the predicted curves by the DVMs, XCDM and the Λ CDM, for the best-fit values in Table I. The highest (dotted) curve in brown corresponds to model Q_Λ for the same best-fit values as [46] and assuming (as they do) that the dark sector interaction begins at $z = 0.9$. Such strayed curve is unable to explain the LSS data (see the text for discussion). Shown are the values of $\sigma_8(0)$ that we obtain for all the models.

from the data (cf. caption of Table I) and include the corresponding covariance matrices. See Appendix B of [42].

Discussion: We find (cf. Table I and Fig. 1) that the effective vacuum parameters ν_i of each DVM are projected non null and positive, attaining 3.76σ and 3.60σ c.l. for both ν (RVM) and ν_{dm} (model Q_{dm}), respectively. For model Q_Λ , however, we find $\nu_\Lambda > 0$ at 2.38σ c.l. For the XCDM parametrization we find a departure of the dark energy EoS from the value $w = -1$ at 3.35σ c.l. into the quintessence regime ($w \gtrsim -1$). As indicated above, similar results to the XCDM are obtained for models with a mildly evolving EoS, such as the CPL parametrization, or even with specific scalar fields with a given potential [42, 51]. We find this fact noteworthy.

The χ^2_{\min} -value of the overall fit for each DVM and the XCDM is clearly smaller than the Λ CDM one (cf. Table I). For a better assessment of the situation, it proves very useful to invoke the time-honored Akaike and Bayesian information criteria, AIC and BIC, see [53, 54]: $AIC = \chi^2_{\min} + 2nN/(N - n - 1)$ and $BIC = \chi^2_{\min} + n \ln N$, where n is the number of fitting parameters and N the number of data points. These criteria are useful to analyze competing models describing the same data [55]. The larger are the differences ΔAIC (ΔBIC) with respect to the model that carries smaller value of AIC (BIC) – the DVMs and XCDM here – the higher is the evidence against the model with larger value of AIC (BIC) – the Λ CDM. For ΔAIC and/or ΔBIC in the range 2 – 6 we can speak of “positive evidence” while for the range 6 – 10 we can speak of “strong evidence” against the Λ CDM, and hence in favor of dynamical DE. Above 10, we are entitled to claim “very strong

evidence” [53–55] against the Λ CDM. Table I renders the net results $\Delta AIC > (12.9, 12.1, 8.5, 3.4)$ and $\Delta BIC > (10.6, 9.8, 6.3, 1.1)$ for the four DE models under study (RVM, Q_{dm} , XCDM, Q_Λ). Thus, two DVMs (RVM and Q_{dm}) and the XCDM are definitely more favored than the Λ CDM, and the most conspicuous one is the RVM (5). As for Q_Λ , AIC and BIC indicate “moderate evidence” in its favor. These results are consistent and outstanding since *both* AIC and BIC peak strongly (or very strongly) in the same direction.

Full details of our analysis can be found in the comprehensive companion paper [42]. There we show the fitting results that we obtain under a variety of situations, including a discussion of the cosmological data used and a thorough study of the impact from the different data sources on the overall fit. We find that the dynamical DE effect reported here mainly emerges from the crucial intersection of the triad of BAO+LSS+CMB data. Depending on the analysis of the errors involved in some of the BAO+LSS data (cf. e.g. Ref.[22, 56] and references therein) the effect can be slightly more pronounced than the one reported here, but not below it. In all cases we find that the DE dynamics for the main DVMs (RVM and Q_{dm}) is confirmed at a confidence level in between $3.6 - 4.3\sigma$, see [42]. These results are, in addition, backed up with compelling ΔAIC and ΔBIC values firmly anchored in the high ranges 12 – 18 and 10 – 15 respectively [42], thereby ensuring “very strong evidence” (according to the conventional usage of these information criteria [55]) of the claimed effects. Remarkably enough, significant support at 3.5σ c.l. for dynamical DE from the observational point of view has appeared in the literature very recently [57].

Conclusions: The hypothesis $\Lambda = \text{const.}$ despite being the simplest may well not be the most favored one at present when we put it in hard-fought competition with specific dynamical DE models. Upon performing a thorough analysis of a rich and fully updated sample of SNIa+BAO+ $H(z)$ +LSS+CMB data we conclude the following. In terms of the simple XCDM parametrization we find unprecedented $> 3.3\sigma$ evidence of “effective” quintessence behavior. Phrased in terms of the DVMs, the evidence of $\rho_\Lambda \neq \text{const.}$ is enhanced up to $\sim 4\sigma$ for the best two models. These signs of dynamical DE are riveted with the firm verdict of Akaike and Bayesian criteria. We conclude that the current cosmological data do uphold in a rather significant way a mild evolution of the DE, in contrast to the concordance model. We hope that this result might inject some more optimism for an eventual solution of the hard cosmological constant problem and the associated cosmic coincidence problem.

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